Problem Set I: Recurrent Problems

October 4, 2025

Instructions: No need to solve them fully, just ponder them. We'll probably go over the solutions of a few of them during class. Have fun!

Problem 1. (Knuth, Ch1P8)
Solve the recurrence

$$\begin{cases} Q_0 = \alpha \\ Q_1 = \beta \\ Q_n = \frac{1+Q_{n-1}}{Q_{n-2}} \text{ , for } n > 1. \end{cases}$$

You can assume that $Q_n \neq 0$ for all $n \geq 0$. Remember: it's always a good idea to check small cases first. How would you go about proving your conjecture?

Problem 2. (Knuth, Ch1P14)
Consider a 3-dimensional generalization of the *Lines in the Plane* problem we solved, where we established that the maximum number of regions $\mathcal{L}_n$we can define in the plane using n lines is given by

$$\mathcal{L}_n = \frac{n(n+1)}{2} + 1.$$

What is the maximum number of three-dimensional regions $\mathcal{P}_n$ we can define in 3D space using n planes?

Problem 3. (Knuth, Ch1P6)
Notice that when defining regions in the plane using lines, some regions extend to infinity and some are bounded (say, they have finite area). What is the maximum number of *bounded* regions $\mathcal{B}_n$ we can define using n lines?